Implementing Distributed Control on Star Architectures

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Introduction

**Synthesis**
Given a specification, construct a “plant” that satisfies it (writing correct programs).

**Control**
More general: given a plant and a specification, construct a “controller” so that the plant + controller satisfies it.

We study control on Distributed Systems.
Distributed Models

There is no canonical model. Many powerful ones, like communicating automata, but even the simplest problems (eg. reachability) are undecidable.

We consider Zielonka or Asynchronous automata.

Why Zielonka Automata?

- Closely linked to Mazurkiewicz traces - have rich theory, generalizing results on words.
- High level model of synchronization.
- Simple: no buffers (Turing Powerful).
Deterministic Zielonka (Asynchronous) Automata

\[ A = \langle \{ S_p \}_{p \in P}, s_{in}, \{ \delta_a \}_{a \in \Sigma} \rangle \]

- Finite set of processes: \( P = \{ p, p' \} \).
- Distributed alphabet: \( \Sigma \) finite, 
  \( \text{dom} : a \mapsto \{ p \}, \ c \mapsto \{ p, p' \} \)
- Finite set of (local) states: 
  \( S_p = \{ s_0, s_1, s_2 \}, \ S_{p'} = \{ s'_0, s'_1, s'_2 \} \).
- \( s_{in} = (s_0, s'_0) \).
- Deterministic transition functions:
  \[
  \delta_c : S_p \times S_{p'} \rightarrow S_p \times S_{p'} \\
  (s_1, s'_1) \mapsto (s_0, s'_0) \\
  (s_2, s'_2) \mapsto (s_0, s'_0)
  \]

\[ L(A) = \left( (\text{shuffle}(ab) + \text{shuffle}(aabb)) \cdot c \right)^* \cdots \]
Order of execution of \(a, b\) irrelevant. Independence relation \(I\).

\((a, b) \in I\) iff \(\text{dom}(a) \cap \text{dom}(b) = \emptyset\). Induces congruence \(\sim_I\) on \(\Sigma^*\).

Trace: an equivalence class \([w]_I\). \(L(A)\) is trace-closed.

**Zielonka’s Theorem (’87)**

Every regular trace-closed language can be recognized by a deterministic Zielonka automaton (of exponential size in \(\mathbb{P}\), Genest et al. ’09).
Control Problem

Variant of the control problem in [Ramadge and Wonham, '89].

Given

- Distributed alphabet, but partitioned: $\Sigma = \Sigma^{sys} \cup \Sigma^{env}$
- Deterministic Zielonka automaton $P$ (the plant)

Find

- Another Zielonka automaton $C$ over the same alphabet (i.e. same processes) such that the product $P \times C$ satisfies a given specification.
- $C$ cannot block environment actions.

Assuming

- All environment actions are local.
- All synchronization actions are binary: $\forall a \in \Sigma, |dom(a)| \leq 2$. 
Finding a controller equivalent to finding a distributed strategy in a game between system and environment.

**The Game**

- Arena: Zielonka automaton $\mathcal{A}$.
- At each step: each process proposes a set of actions, environment chooses which to execute.
- A play is a trace from $L(\mathcal{A})$ (finite/infinite).
- Strategy: $\sigma = (\sigma_p)_{p \in P}$, where $\sigma_p : \text{Plays}_p(\mathcal{A}) \rightarrow 2^{\Sigma_p^{sys}}$.
- The set of $\sigma$-plays is defined to contain $\epsilon$, and for all $\sigma$-plays $t$:
  - if $a \in \Sigma^{env}$, $ta \in L(\mathcal{A})$ then $ta$ is a $\sigma$-play;
  - if $a \in \Sigma^{sys}$, $ta \in L(\mathcal{A})$ and $a \in \sigma_p(t)$ for all $p \in \text{dom}(a)$ then $ta$ is a $\sigma$-play.
Problem Statement

Winning Condition

- Stated on maximal plays.
- Local reachability condition: Each process has $F_p \subseteq S_P$. We assume that final states are absorbing, i.e. if $s_p \xrightarrow{a} s'_p$ and $s_p \in F_p$ then $s'_p \in F_p$.

Equivalence

It is easy to see that a controller defines a strategy. Conversely, a winning strategy can be transformed to a correct controller (hard).

Distributed Control Problem

Given a plant (Zielonka automaton) $A$, and a local reachability condition $(F_p)_{p \in P}$ determine if there is a distributed strategy $\sigma = (\sigma_p)_{p \in P}$ such that every maximal $\sigma$-play $t$ eventually ends in $F_p$ for all $p$. 
Results

Decidability is open in the general case.

- Previous Results:
  - [Madhusudhan & Thiagarajan 2002] Decidable with strong restriction on local strategies.
  - [Madhusudhan & Thiagarajan 2005] Decidable with restriction on plant.

Related Work
Define communication graph on $P$ with edge $\{p, q\}$ if there exists $a \in \Sigma$ with $\text{dom}(a) = \{p, q\}$.

- [Genest & Gimbert & Muscholl & Walukiewicz] If the communication graph is acyclic, the control problem is decidable (non-elementary complexity). Assumes final states are blocking.
In This Talk:

Special case:

- Communication graph a tree of depth 1.
- Real world: client-server architecture.
- EXPTIME complexity algorithm to decide and recover strategy.
- Construction generalizes to other winning conditions (non-blocking, Büchi).
Example

\[ \delta_# : (xX, yY) \mapsto (s_1, s'_1) \iff x = y \text{ or } X = Y. \]
Basic idea: reduce distributed game $\mathcal{A} \rightarrow$ sequential reachability game $\mathcal{A}'$.

Root process $q$ “simulates” child processes $r_1, \ldots, r_k$.

Key lemma: can simulate actions of a child between two synchronizations by a \textit{positional strategy}.
Construction of $A'$

\[
\text{Root} \langle s_q, (s_j, \tau_j)_{1 \leq j \leq k} \rangle
\]

- $s_q \in S_q, s_i \in S_{r_i}, \tau_i$ local positional strategy for $r_i$
Construction of $A'$

- $s_q \in S_q$, $s_i \in S_{r_i}$, $\tau_i$ local positional strategy for $r_i$
Environment either chooses a local action $a$ for $q$

- $s_q \in S_q$, $s_i \in S_{r_i}$, $\tau_i$ local positional strategy for $r_i$
Construction of $\mathcal{A}'$

Or chooses a synchronization action $b$ between $q$ and $r_i$.

- $s_q \in S_q$, $s_i \in S_{r_i}$, $\tau_i$ local positional strategy for $r_i$
Construction of $A'$

- $s_q \in S_q$, $s_i \in S_{r_i}$, $\tau_i$ local positional strategy for $r_i$
Construction of $\mathcal{A}'$

- $s_q \in S_q$, $s_i \in S_{r_i}$, $\tau_i$ local positional strategy for $r_i$
- Good $\tau_i$ : every infinite local play from $s_i$ ultimately in $F_{r_i}$.
Construction of $A'$

- $s_q \in S_q$, $s_i \in S_{r_i}$, $\tau_i$ local positional strategy for $r_i$
- Good $\tau_i$: every infinite local play from $s_i$ ultimately in $F_{r_i}$.
- Final states: $s_q \in F_q$, all local maximal finite $\tau_i$-plays from $s_i$ end in $F_{r_i}$.
Proof

**Theorem**

*The system has a winning strategy for $A, (F_p)_{p \in P}$ if and only if the system has a winning strategy for $A', F$.*

**Recovering Strategy**

- $q$ plays the actions as given by $\text{choice}(a), \text{choice}(a, s'_i)$, etc.
- Each $r_i$ plays according to $\tau_i$. 
Implementation Demo

- Implemented as an extension to a Java program called GAVS+, by Chih-Hong Cheng, TU München.
Conclusion

▶ Generalize to other winning conditions: Büchi. Ongoing, have preliminary results.
▶ Other research directions.
Thank you :)}